sondern auch bei großen Elektronendichten zusammen mit hohen Temperaturen und Ionenladungszahlen eine schlechte Näherung der Ionisationsgleichung (2) dar.

Dadurch wird der beschränkte Anwendungsbereich der beiden bekannten Ionisationsgleichungen deutlich und die Notwendigkeit, das Plasma durch die umfassendere Gl. (2) zu beschreiben, bekräftigt.

Eine systematische Auswertung der Zustandsgrößen sowie der Kontinuumsintensität soll für mehrere Plasmen von Interesse in folgenden Arbeiten unternommen werden.

# Investigation of the Cyclotron Harmonic Radiation from a Plasma in a Magnetic Field

EBERHARD RÄUCHLE

Institut für Hochtemperaturforschung der Technischen Hochschule Stuttgart

and WALTHER HESS

Institut für Gasentladungen und Photoelektronik der Technischen Hochschule Stuttgart

(Z. Naturforschg. **21 a**, 383—390 [1966]; received 14 December 1965)

The microwave emission of the positive column of a low pressure discharge in a magnetic field is investigated by means of a 10 000 Mc/sec radiometer. The radiation intensity as a function of the longitudinal magnetic field shows a number of peaks near the electron cyclotron harmonic frequencies. The measurements are performed for noble gases in the pressure range from 10 to $10^{-2}$ Torr at electron densities varying from $10^9$ to $10^{13}$ cm$^{-3}$. In order to compare the experimental results with a theoretical investigation performed by one of the authors [12], the emission of harmonics was observed in dependence on the electron density. A satisfactory agreement between theory and experiment is shown.

## I. Introduction

The microwave emission of a low pressure plasma in a magnetic field is found to exhibit a number of peaks near the electron cyclotron harmonic frequencies. This phenomenon was first observed by LANDAUER [1] using a PIG discharge. Meanwhile, the emission and absorption resonances near the cyclotron harmonics have been investigated by several workers [2–7] using various techniques as well as various techniques as well as various sources of plasma.

Our investigations reported in this paper are performed on the positive column of a hot cathode discharge in neon, argon, and krypton. Various theoretical explanations of the phenomena observed were given in the past. CANNOBIO et al. [8] and recently STONE et al. [9] showed that the harmonics can be understood by assuming longitudinal waves excited by fast electrons. DAWSON et al. [10] gave another explanation considering a jump in the density distribution. In these mentioned papers [8–10], it had to be assumed that the plasma frequency $\omega_p$ is higher than the electron cyclotron frequency $\omega_c$, in order to obtain an emission of higher harmonics at all. Furthermore a mechanism for the transformation of the longitudinal waves into transverse waves must be introduced additionally [11]. Another explanation, considering the direct excitation of transverse waves in the plasma, was given by one of the authors [12]. In this theory it

1 G. LANDAUER, Proc. 5th Intern. Conf. Ionization Phenomena in Gases, North-Holland Publishing Co., Amsterdam 1962, Vol. 1, p. 389; J. Nucl. Energy **4**, 395 [1962].
2 G. BEKEFI, J. D. COCCOLI, E. B. HOOPER, JR., and S. J. BUCHSBAUM, Phys. Rev. Letters **9**, 6 [1962].
3 F. W. CRAWFORD, G. S. KINO, and H. H. WEISS, Phys. Rev. Letters **13**, 229 [1964].
4 S. TANAKA and H. KUBO, Inst. of Plasma Physics Report, IPPJ-23, Nagoya University, Nagoya, Japan 1964.
5 C. D. LUSTIG, W. D. MCBEE, and A. KALISKY, Rev. Sci. Instr. **35**, 869 [1964].
6 C. D. LUSTIG, Phys. Rev. **139**, A 63 [1965].
7 W. HESS and E. RÄUCHLE, Proc. 7th Intern. Conf. Phenomena in Ionized Gases, 1965 (to appear), paper Nr. 4.5.1.(10).
8 E. CANOBBIO and R. CROCI, Proc. 6th Intern. Conf. Ionization Phenomena in Gases, Paris 1963, Vol. 3, p. 269.
9 P. M. STONE and P. L. AUER, Phys. Rev. **138**, A 695 [1965].
10 J. M. DAWSON and A. F. KUCKES, Phys. Fluids **8**, 1007 [1965].
11 T. H. STIX in: Annual Report MATT-Q-22, Princeton University, Princeton 1965, p. 307.
12 E. RÄUCHLE, Inst. Hochtemperaturforschung d. Techn. Hochschule Stuttgart, Report Nr. 3—5, 1964; Proc. 7th Intern. Conf. Phenomena in Ionized Gases, 1965 (to appear), paper Nr. 4.5.1 (3).

is not necessary to assume a limiting electron density condition as well as no transformation mechanism must be introduced.

The following investigation was undertaken in order to test the various explanations for the observed resonances. In this connection we were especially interested in the dependence of the measured harmonic intensity on the electron density. We have found that the resonances also appear at low electron densities. In these cases, the plasma frequency is lower than the frequency of radiation.

## II. Experimental Arrangement

### *1. The Discharge Tube*

The weakly ionized plasma is generated by a hot cathode discharge in a quartz tube of 20 mm diameter and 50 cm length, filled with neon, argon or krypton at a pressure varying from 10 to $10^{-2}$ Torr. Before filling the tube with the noble gas it was prepared on a vacuum system as usual. The dc tube current $I_e$ was varied from $10^{-3}$ to 3.0 amp., yielding electron densities in the range of $10^9$ to $10^{13}$ $cm^{-3}$. The tube is placed coaxially in the center of a solenoid with a maximum magnetic field of 3.4 kG, as shown in Fig. 1. The electrodes of the tube are situated outside the magnetic field.

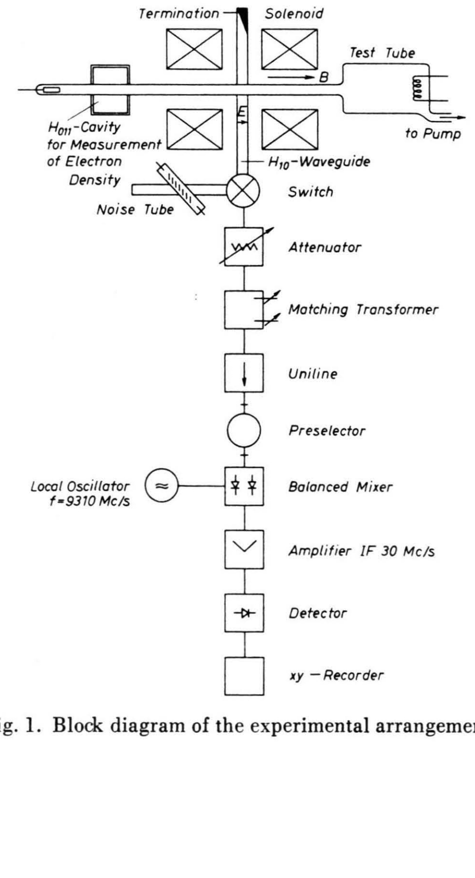


Fig. 1. Block diagram of the experimental arrangement.

### *2. Radiation Measurements*

A part of the positive column of the discharge is located within a rectangular $H_{10}$-waveguide, the axis of which is perpendicular to the magnetic field. We have observed the ordinary wave, that is, the rf electric vector in the guide was orientated parallel to the magnetic field. This orientation was selected because we are especially interested in the emission of cyclotron harmonics and not in an investigation of the hybrid resonance radiation [13]. A further advantage of this orientation is its immediate use for electron density measurements (see below).

The microwave radiation propagating down the waveguide was measured by means of a 10 Gc/sec radiometer of the superheterodyne type. The receiver shown in Fig. 1 with an overall noise figure of about 8 dB is operated at a fixed frequency of $\omega/(2\pi) =$ 9340 Mc/sec with a 4 Mc/sec bandwidth $\Delta f$. It was calibrated against a noise standard of known radiation temperature. Power levels down to $10^{-14}$ w could be measured with the radiometer shown in Fig. 1. In some cases it was necessary to increase the sensitivity of the receiver down to $10^{-16}$ w. This was achieved by the use of phase sensitive detection. For this purpose the microwave intensity was modulated in amplitude by the diode switch and the detector in Fig. 1 was followed by a lock-in-amplifier to demodulate the received power.

In both cases the radiation pattern, that is the radiometer output as a function of the magnetic field, was displayed on a recorder.

### *3. Electron Density Measurements*

Recording the radiation patterns for various plasma parameters such as neutral gas pressure, discharge current and the chemical nature of the noble gas used in the test tube, we were especially interested in an accurate determination of the electron density. Hence the following three different methods were used for these measurements:

The first was a cavity method [14] giving the average electron density of the emitting low pressure column by means of the frequency detuning of a $H_{011}$-cavity, resonating at 10 Gc/sec. The cavity is located outside the magnetic field in front of the anode of the tube as shown in Fig. 1.

The second method consists of the measurement of the noise power radiated from the discharge with zero magnetic field as a function of the discharge current. As BEKEFI et al. [15] have shown for the case of electron-

[13] J. L. HIRSHFIELD and S. C. BROWN, Phys. Rev. **122**, 719 [1961].

[14] M. A. BIONDI and S. C. BROWN, Phys. Rev. **75**, 1700 [1949]. — See also H. J. OSKAM, Philips Res. Rep. **13**, 335 [1958]; W. HESS, Phys. Letters **12**, 211 [1964]; Z. Angew. Phys. **18**, 68 [1964]; Z. Naturforschg. **20 a**, 451 [1965].

[15] G. BEKEFI, J. L. HIRSHFIELD, and S. C. BROWN, Phys. Rev. **116**, 1051 [1959].

atom collisions, the dependence of the noise power $P$ on electron density $n_e$ falls into three regions:

a) The transparent region with a linear increase of the noise power $P$ with electron density;

b) the semiopaque region with a nonlinear increase that occurs in the vicinity of $(\omega_p/\omega)^2 = 1 - (\lambda/\lambda_c)^2$ ($\lambda_c$ is the cutoff wavelength of the waveguide), and

c) the opaque region. In this region the noise power is essentially density independent.

The transition from the transparent region to the opaque region with increasing discharge current then gives a correlation between the discharge current and the plasma frequency.

The cavity method and the noise power radiation method essentially agreed in the determination of the electron density. It is to be noted that both methods give the average electron density of the plasma column outside the magnetic field and that all the data of electron density given in terms of $(\omega_p/\omega)^2$ in the next sections refer to these measurements with zero magnetic field.

For the determination of the electron density in the presence of the magnetic field, a third method was applied. It consists of the measurement of the power absorbed by the plasma. For this purpose the termination in Fig. 1 was replaced by a signal generator feeding a constant microwave power of $10^{-6}$ w into the plasma. The absorption of the plasma could then be inferred from the power transmitted through the plasma indicated by the receiver.

## III. Results of Measurements

### *1. Electron density measurements*

The noise power $P$ radiated from a krypton plasma with zero magnetic field at various neutral gas pressures is plotted in Fig. 2 as a function of the discharge current $I_e$. We note that in this typical example the dependence of the noise power on the electron density falls into three regions described in the previous section. Along the lower abscissa the electron density is plotted in terms of $(\omega_p/\omega)^2$ as measured by the cavity method in a $5 \cdot 10^{-2}$ Torr krypton plasma (curve c). The vertical dashed line at $(\omega_p/\omega)^2 = 1 - (\lambda/\lambda_c)^2$ meets curve c in Fig. 2 in the semiopaque region [15], showing the good agreement of both measuring methods.

When the plasma column is within a longitudinal magnetic field, the axial electron density is increased and the radial electron density distribution is altered. The experimental study of BICKERTON et al. [16] has shown, e. g., that in a helium plasma ($p = 0.048$ Torr) the axial electron density is increased by a factor of

[16] R. J. BICKERTON and A. VON ENGEL, Proc. Phys. Soc. London 69 B, 468 [1956].

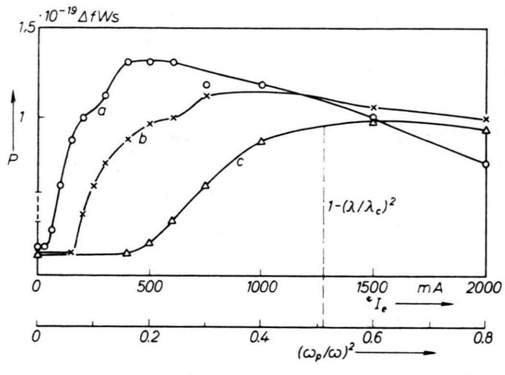


Fig. 2. Noise power $P$ from a krypton plasma as a function of the discharge current $I_e$ for various neutral gas pressures with zero magnetic field. Curve a: 0.5 Torr, b: 0.1 Torr, c: 0.05 Torr. The normalized electron density $(\omega_p/\omega)^2$ plotted along the lower abscissa belongs to curve c.

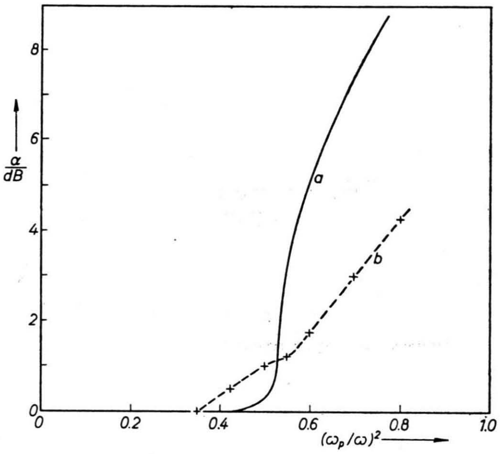


Fig. 3. Wave attenuation $\alpha$ for a plasma-filled waveguide as a function of the electron density $(\omega_p/\omega)^2$. a: calculated for $\nu/\omega = 10^{-3}$ ($\nu$ = electron-atom collision frequency), b: measured in a neon plasma at a neutral gas pressure of 0.35 Torr.

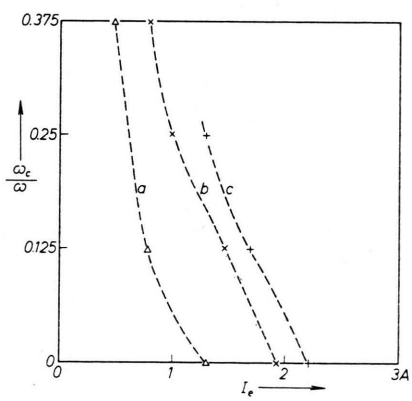


Fig. 4. Dependence of the normalized magnetic field $\omega_c/\omega$ on the discharge current $I_e$, which is necessary to produce an electron density of $0.4(\omega_p/\omega)^2$ in a neon plasma. Curve a: 0.35 Torr, b: 0.2 Torr; c: 0.1 Torr.

3 when the magnetic field is raised from 0 to 500 G, corresponding to the 6th harmonic in our case.

To determine this increase in electron density in our experiments, the microwave power absorbed by the plasma in the magnetic field was measured. Fig. 3 shows the attenuation $\alpha$ calculated from the theory for plasma-filled waveguides [17] as well as a measured attenuation curve as a function of the electron density. In the case of orientation of the electric, the magnetic, and the propagation vectors as described above, the wave propagation is not directly influenced by the magnetic field. The only influence of the magnetic field consists of the increase of the electron density in the plasma column. From Fig. 3 follows that the onset of absorption near $(\omega_p/\omega)^2 \approx 0.4$ is very sensitive on electron density. For this reason we have investigated this onset of absorption for different magnetic fields, giving a correlation between the magnetic field and the discharge current necessary to produce a normalized electron density of $(\omega_p/\omega)^2 \approx 0.4$. This is shown in the plot of Fig. 4 for different gas pressures in a neon plasma. From Fig. 4 follows that the electron density of a 0.1 Torr neon plasma at a normalized magnetic field of $\omega_c/\omega = 8^{-1}$ [8th harmonic, $\omega_c = e B/(m c)$] is increased by a factor of 1.25; at $\omega_c/\omega = 4^{-1}$ (4th harmonic) the factor is 1.65; for the third harmonic it is of the order of 2.0.

### 2. *Radiation measurements*

In Fig. 5, three examples of characteristic radiation patterns $N(\omega_c/\omega)$ are given for neon and argon. It is seen from this plot that a number of small resonant peaks near the cyclotron harmonics $\omega_c/\omega = 1/n$ are superimposed on an emission background. To explain this background radiation we refer back to Fig. 2, where the noise power increases with increasing electron density until the plasma reaches the opaque region near $(\omega_p/\omega)^2 \geqq 1$. In Fig. 5 the situation is similar, with one difference: the measurements shown in Fig. 2 were performed with zero magnetic field and the electron density was increased by raising the discharge current. On the other hand, the plots in Fig. 5 were recorded at a constant discharge current but with increasing magnetic field. As we have shown in the previous section, the magnetic field is responsible for an increase in electron density, depending on the magnitude of the magnetic field. Considering curve „a“ in Fig. 5, it is seen that in the region $0 < \omega_c/\omega < \frac{1}{4}$ the plasma is transparent, whereas for higher magnetic fields it reaches the opaque limit. In the range $\frac{1}{10} < \omega_c/\omega < \frac{1}{3}$, the electron density is increased by a factor of 2 from $(\omega_p/\omega)^2 = 0.35$ to 0.7. Curve „c“ in Fig. 5 shows a steeper increase of the background radiation, consistent with the electron density measurements for an argon plasma. The decrease of the radiation intensity at higher magnetic fields is probably caused by an instability or by a quenching of the plasma column.

As to the resonant peaks, it is seen that the amplitudes of the harmonics are small compared to the background radiation.

In Fig. 6, a comparison of the radiation pattern of an initially (for zero magnetic field) transparent and an opaque plasma is shown. Curve „a“ in Fig. 6 is similar to the plots „a“ and „b“ in Fig. 5 (please note the other scale), while curve „b“ shows the radiation pattern of a krypton plasma, which is opaque $[(\omega_p/\omega)^2 \approx 5.0]$, even for zero magnetic field. We can see from this plot that near the third and the second harmonic, the emission line dips below the background radiation (see also Ref. [4]). The exhibited decrease of the background radiation at higher magnetic fields is consistent with the decrease of the electron temperature with increasing magnetic field [18].

In the following we shall summarize our main results:

a) In the pressure range from 10 to 0.5 Torr the collision frequency of the plasma was too high for an observation of the cyclotron harmonics of higher order. Neither in a transparent nor in an opaque plasma, the harmonics could be found (in this case there is a strong collision broadening). The inverted line of the second and in some cases of the third harmonic dips below the background radiation for all investigated gases.

b) In the low pressure range from $10^{-1}$ to $10^{-2}$ Torr the peaks up to the 10th harmonic could be observed. All the measurements were performed in the transparent or semiopaque region of the plasma. To obtain the real intensity of the harmonics, the electron-atom collision radiation background was subtracted from the measured radiation patterns. In

[17] M. A. HEALD and C. B. WHARTON, Plasma Diagnostics with Microwaves, John Wiley, New York 1965, p. 166.

[18] S. C. BROWN, Basic Data of Plasma Physics, Technology Press of the MIT, Cambridge, Massachusetts, 1959, p. 293.

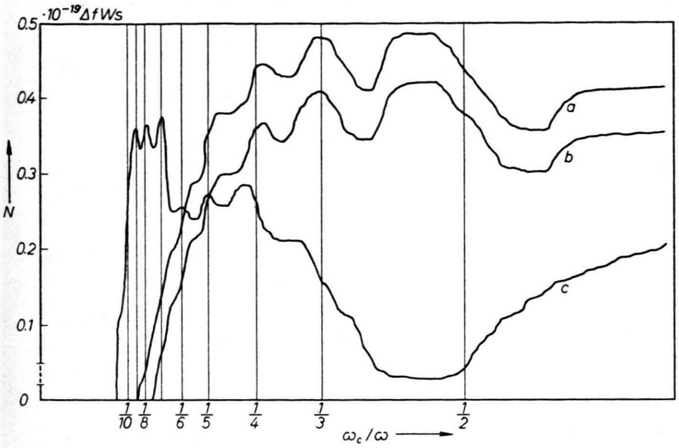


Fig. 5. Microwave power $N$ radiated as a function of the normalized magnetic field $(\omega_c/\omega)$ in the low pressure range. Curve a: neon, 0.08 Torr, $(\omega_p/\omega)^2=0.35$; b: neon, 0.08 Torr, $(\omega_p/\omega)^2=0.25$; c: argon, 0.04 Torr, $(\omega_p/\omega)^2=0.3$.

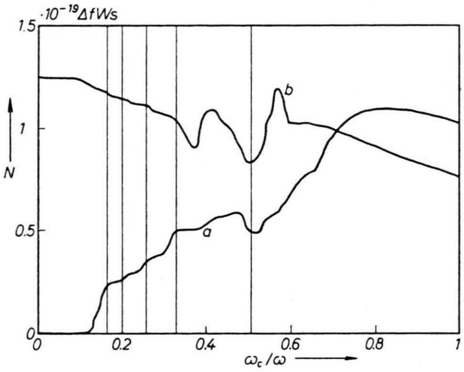


Fig. 6. Microwave power $N$ radiated as a function of the normalized magnetic field $(\omega_c/\omega)$ for a transparent plasma (curve a) and an opaque plasma (curve b). Curve a: neon, 0.06 Torr, $(\omega_p/\omega)^2=0.2$; b: krypton, 0.5 Torr, $(\omega_p/\omega)^2=5.0$.

Fig. 7 the harmonic intensity $I$ emitted from a neon plasma with an initial electron density of $(\omega_p/\omega)^2 = 0.3$ is given as a function of $\omega/\omega_c$. This plot was taken from a recorded radiation pattern $N(\omega_c/\omega)$ by inverting the scale of the abscissa and by reducing the measured radiation by the background emission. Fig. 7 shows that the radiation intensity of the successive peaks decreases with decreasing magnetic field, if the frequency of observation is considered constant. This decrease in intensity is caused by two effects which cannot directly be separated in our experiments: The first is a reduction in electron density as the magnetic field is decreased to record the higher harmonics; the second is predicted by the theory of the magnetic bremsstrahlung [12], because the frequency integrated intensity is proportional to the magnetic field intensity.

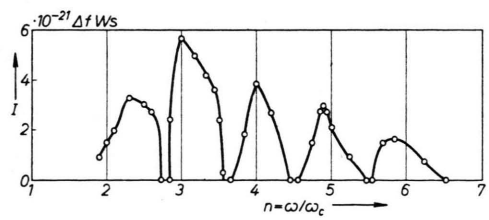


Fig. 7. Radiation intensity $I$ of the harmonics as a function of the harmonic number $n=\omega/\omega_c$ for a neon plasma ($p=0.06$ Torr, $(\omega_p/\omega)^2=0.3$).

c) As to the dependence of the harmonic intensity on the initial electron density, Fig. 8 shows that it gradually increases with increasing electron density.

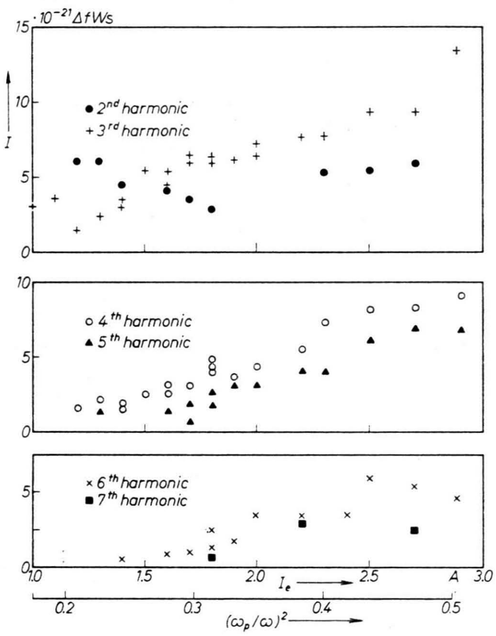


Fig. 8. Radiation intensity $I$ of the first 7 harmonics as a function of the normalized initial electron density $(\omega_p/\omega)^2$ for a neon plasma with a neutral gas pressure of 0.06 Torr.

Please note that the $(\omega_p/\omega)^2$-scale in Fig. 8 must be multiplied by a constant factor in order to obtain the real electron density at a discrete harmonic. For the third harmonic, e. g., we have shown in Section 1 that the factor is of the order of 2.0. Considering this, it is seen from Fig. 8 that all the measurements were carried out in the electron density range

$(\omega_p/\omega)^2 < 1$. The highest intensity is found for the third harmonic to be of the order of $I \approx 10^{-20}$ w/c sec.

The plot of the amplitudes of the first seven resonances for an argon and a krypton plasma of low gas pressure is similar to that shown in Fig. 8. For all investigated gases it is further found that the magnitude of the gas pressure has a small influence on the intensity of a discrete harmonic, provided the pressure is low enough ($p < 0.1$ Torr).

## IV. Discussion of Results

### *1. Comparison with previous experiments*

We compare the experimental results described above with our theory of the bremsstrahlung in a magnetic field [12] as well as with experiments carried out recently by LUSTIG [6] and others. The measurements [6] performed with an argon plasma of $10^{-3}$ Torr try to confirm the theory of STONE et al. [9], considering the excitation of electrostatic waves as a possible mechanism to understand the cyclotron harmonic radiation.

The experiments of LUSTIG seem to disagree with our measurements in the following points:

a) LUSTIG has found a strong dependence of the harmonic intensity on electron density. If the electron density is raised by a factor of about 2, the harmonic intensity varies from $10^{-19}$ to $10^{-16}$ w/c sec. On the other hand we have measured an intensity variation from $10^{-21}$ to $10^{-20}$ w/c sec only. The amount of the harmonic radiation was smaller by four orders of magnitude. Similar low intensities were also found by TANAKA et al. [4].

b) LUSTIG did not observe any radiation unless the electron density was large enough, that is in the range of $(\omega_p/\omega)^2 \geqq 1$. Our experiments were performed in a transparent or semiopaque plasma in the electron density range $(\omega_p/\omega)^2 < 1$.

c) As to the measured spectrum, LUSTIG reports that it consists of three lines near each harmonic. We could not observe this splitting of the lines.

We infer the above discrepancies from the existence of different classes of phenomena, depending on the parameters of the plasma. We feel that in one case [6] the high intensity emission is radiated from a turbulent plasma and is influenced by the axial gradients of the electron density as well as the magnetic field [19].

In another experiment [20], radiation intensities as high as 500 w at a frequency of 10 Gc/sec were measured in a PIG-discharge with an operation pressure of $10^{-3}$ Torr. A magnetic mirror field was applied to this discharge.

A rough estimate of the critical magnetic field $B_c$ responsible for a plasma instability as derived by KADOMTSEV et al. [21] gives a value of $B_c \approx 100$ G, assuming an argon plasma with a neutral gas pressure of $10^{-3}$ Torr. This value is within the region, in which the experiments of LUSTIG [6] and WANIEK et al. [20] were carried out. The sudden rise of the intensity in a small electron density region measured by LUSTIG seems to prove the existence of plasma turbulence in his experiments. On the other hand, the measurements reported in the present paper are performed away from the axial magnetic field gradient as well as below the critical magnetic field $B_c$ in an axially homogeneous plasma region. From this reason we believe that in our case no turbulence mechanism will be responsible for the excitation of the harmonics. We can further show that the observed small radiation intensities that are one or two orders of magnitude below the thermal noise radiation of the opaque plasma (see Figs. 7 and 8) can be explained with the theory of the magnetic bremsstrahlung [12]. In the last section, we compare our experimental results with this theory.

### *2. Comparison with the theory of bremsstrahlung in a magnetic field*

An important fact is that the harmonics could be found in a transparent plasma at low electron densities $(\omega_p/\omega)^2 < 1$. At these low densities the other explanations [9–11] considering longitudinal waves exhibit no harmonics; furthermore the transformation of longitudinal waves into transverse waves of the extraordinary mode as proposed by STIX [11] is not possible at low electron densities [22].

[19] See Fig. 7 and Fig. 8 in ref. [6]. The measurements reported in ref. [6] are performed in the length of the plasma column that was near the field- and electron density gradient.

[20] R. W. WANIEK, R. T. GRANNAN, and D. G. SWANSON, Proc. 7th Intern. Conf. Phenomena in Ionized Gases, 1965 (to appear), paper Nr. 4.5.1.(15).

[21] B. B. KADOMTSEV and A. V. NEDOSPASOV, J. Nucl. Energy **1**, 230 [1960]. — K. H. WÖHLER, Z. Naturforschg. **17 a**, 937 [1962].

[22] Another transformation mechanism considering ion density fluctuations was proposed by T. J. M. BOYD, Proc. 7th Intern. Conf. Phenomena in Ionized Gases, 1965 (to appear), paper Nr. 4.5.1.(7).

STIX assumed that there exists an electron density inhomogeneity in the plasma where the wave frequency equals the upper hybrid frequency. In our experiments described above, the upper hybrid frequency was smaller than the measuring frequency; at the same time we did not detect the extraordinary wave polarization but the ordinary wave polarization.

In our theory for the direct emission of transverse radiation by bremsstrahlung in a magnetic field, intensity maxima are obtained near harmonics of the cyclotron frequency. The mechanism responsible for this bremsstrahlung is the interaction of gyrating particles with the statistical fluctuations of the plasma potential (plasmon field). Calculating the radiative emission from accelerated charged particles in a volume element $\mathrm{d}V$, in the frequency range $\mathrm{d}\omega$, and into the solid angle $\mathrm{d}\Omega$, there results for the differential intensity distribution

$$\frac{\mathrm{d}I}{\mathrm{d}V\,\mathrm{d}\Omega\,\mathrm{d}\omega} = \frac{e^4\,n_\mathrm{e}}{(2\,\pi)^4\,c^3\,m^2} \lim_{V,\tau\to\infty} \frac{1}{V\,\tau}\,(\delta_{rs} - \mathfrak{e}_r\,\mathfrak{e}_s) \cdot \sum_n \int_k k_r\,k_s\,\Phi^*(p,\boldsymbol{k})\,\Phi(p,\boldsymbol{k})\,J_n^2\left(\frac{K\,v}{\omega_\mathrm{c}}\right)\mathrm{d}^3\boldsymbol{k}. \tag{1}$$

$\Phi(p,\boldsymbol{k})$ is the spectral distribution of the plasma potential, $J_n$ the BESSEL function of $n$-th order, $\boldsymbol{k}$ the wave vector with Cartesian components $k_r$, forming a component $K$ perpendicular to the magnetic field, $V$ the plasma volume, $\tau$ the time of observation, $\mathfrak{e}$ (with components $\mathfrak{e}_r$) the direction of observation, $m$ is the mass of the accelerated test electron, $v$ and $v_z$ are its velocities perpendicular and parallel to the magnetic field,

$$p = n\,\omega_\mathrm{c} + k_z\,v_z - \omega$$

is the difference between the emitted frequency and the effective frequency of the rotating electron.

In deriving Eq. (1) it was assumed, that the accelerated electron radiates as in vacuum. This means that the transverse dielectric constant $\varepsilon_\mathrm{t}$ was set equal to unity. This is possible in a wide frequency range [23] for the ordinary wave propagation and for low temperatures and electron densities existing in the positive column. By considering polarization effects, $\varepsilon_\mathrm{t} \neq 1$, the radiative emission would be further increased in the immediate neighbourhood of the harmonics. Eq. (1) is not necessarily restricted to thermodynamic equilibrium. An emission from nonequilibrium plasmas will be treated in a separate paper [24]. If the FOURIER transform of the fluctuating electrical potential is known, the spectral emission intensity can be calculated. One can obtain this FOURIER transform in different ways, e. g. in a first approximation by a test particle assumption. For the most general case only numerical solutions for the intensity distribution are available.

The frequency integrated intensity distribution however, can be given analytically. It is identical with the area under the curve of Fig. 7 [25]. For a direction of observation perpendicular to the magnetic field and an electric field vector parallel to the magnetic field we obtain for the frequency integrated intensity distribution

$$\frac{\mathrm{d}I}{\mathrm{d}V\,\mathrm{d}\Omega} \equiv \int_\omega \frac{\mathrm{d}I_z}{\mathrm{d}V\,\mathrm{d}\Omega\,\mathrm{d}\omega}\,\mathrm{d}\omega\,,$$

$$\frac{\mathrm{d}I}{\mathrm{d}V\,\mathrm{d}\Omega} = \frac{2\,e^6\,n_\mathrm{e}^2\,\omega_\mathrm{c}}{c^3\,m^2\,\sqrt{\varkappa\,T/m}} \cdot \sum_n \int_0^{u_\mathrm{max}} \frac{\exp(-u^2)\;I_n(u^2)}{1+\sqrt{1+2\,(\omega_\mathrm{p}/\omega_\mathrm{c})^2/u^2}}\,\mathrm{d}u\,. \tag{2}$$

$I_n$ is the modified BESSEL function of $n$-th order, $T_\mathrm{e}$ the electron temperature. The upper boundary $u_\mathrm{max}$ appears as an upper limit of the plasmon wave number $K$ perpendicular to the magnetic field. The integral is of the order of unity and only weakly dependent on $n$. The $n$-th element of the sum in the above equation is the frequency integrated intensity of the $n$-the harmonic.

A comparison of Eq. (2) for the bremsstrahlung induced by a plasmon field with the expression for the bremsstrahlung produced by electron-ion collisions in a plasma [26] shows that in both cases intensities of the same amount are obtained. The bremsstrahlung from electron-ion collisions is nearly independent of frequency (white spectrum), the differential intensity distribution of the bremsstrahlung induced by plasmons in a magnetic field, however, depends strongly an frequency.

In Eq. (2) there is a quadratic dependence on the electron density as well as a linear dependence on the magnetic field. This agrees qualitatively with the

[23] YU. N. DNESTROVSKII and D. P. KOSTOMAROV, Soviet Phys.-JETP **13**, 986 [1961].

[24] E. RÄUCHLE, to be published.

[25] Please note that in Fig. 7 the magnetic field and not the frequency of observation has been varied.

[26] See Ref. [17], p. 248.

results shown in Figs. 7 and 8. Considering, e. g., the electron density and the magnetic field variation from the 3rd to the 4th harmonic in Fig. 7, there results a ratio of the areas $F_3 : F_4 \approx 1.8$. From Eq. (2) there follows a value of $F_3 : F_4 \approx 2.4$. For other harmonics we have similar agreement.

A direct comparison of the exact numerical values calculated from Eq. (2) with those of the experiments is difficult to perform for different reasons: Eq. (2) gives the intensity for the transparent case and for free space, while in the experiments the radiation was measured inside the waveguide in the transition region from the transparent to the semi-opaque plasma. In this case we must also consider the absorption of the radiation within the plasma and its reflection at the plasma boundary. These corrections are more important for the lower harmonics than for the higher ones. Another complication for a quantitative comparison of the intensity measurements in waveguides is caused by the gain factor which should be taken into account for a waveguide partially filled with plasma. For the numerical calculation we assume an effective solid angle of $2\pi$. Employing Eq. (2) and inserting the experimental parameters, e. g., for the 6th harmonic in Fig. 7 we obtain a value of about $5 \cdot 10^{-22}$ w/c sec for the maximum radiation intensity. For the 5th, the 4th and the 3rd harmonic the values are $6 \cdot 10^{-22}$ w/c sec, $1.2 \cdot 10^{-21}$ w/c sec and $2.3 \cdot 10^{-21}$ w/c sec. These theoretical results agree with the measured values within the accurancy which can be expected from measurements of this type.

In conclusion, we express our gratitude to Prof. Dr. W. KLUGE for his interest in this work, and to the Deutsche Forschungsgemeinschaft for financial support.

# Messung der Druckerhöhung in einem Wasserstoff-Lichtbogen bei überlagertem axialem Magnetfeld

H. F. DÖBELE, R. WIENECKE und S. WITKOWSKI

Institut für Plasmaphysik, Garching bei München

(Z. Naturforschg. **21 a**, 390—398 [1966]; eingegangen am 22. Dezember 1965)

In a cylindrically symmetric hydrogen arc discharge located in an axial magnetic field, pressure is increased in the inner region, where ionization occurs. Measurements were made with initial pressures of 7.5 and 15 torr, with temperatures between 8000 and 12 000 °K, and with magnetic inductions of 8.5 and 12.5 kGauss. A mercury gauge was connected to a small opening in the anode to measure the radial pressure distribution in the plasma. Temperatures and electron densities were determined spectroscopically. The SAHA equation was found to be applicable in this case. Therefore, experimental results can be compared with theory based on the SAHA equation.

In einer stationären, zylindrischen Lichtbogensäule stellt sich ein Gleichgewicht der radialen Diffusionsströme derart ein, daß an jeder Stelle des Bogens der nach außen gerichtete ambipolare Ladungsträgerstrom gleich dem zur Achse hin gerichteten Neutralteilchenstrom ist. Beim Einschalten eines achsenparallelen Magnetfeldes wird die Diffusion der Ladungsträger herabgesetzt, während die der Neutralteilchen zunächst unbeeinflußt bleibt. Der Druck in den achsennahen, heißen Zonen steigt deshalb solange an, bis in einem neuen Gleichgewichtszustand die beiden Diffusionsströme wieder gleich sind.

WIENECKE [1] hat auf diesen Effekt zuerst hingewiesen und die Druckerhöhung, ausgehend von den Bewegungsgleichungen für ein Dreikomponenten-Plasma, für Wasserstoff- und Heliumbögen berechnet. Unter den dort gemachten Voraussetzungen (kein radialer oder azimutaler Massenstrom, Zylindersymmetrie, lokales thermisches Gleichgewicht) erhält man den Druck im Bogen bei vorgegebenem Außendruck und Magnetfeld als reine Temperaturfunktion. Der Druck hängt also insbesondere nicht vom Radius oder vom Temperaturprofil des Bogens ab. Die Rechnungen zeigen, daß der Druck, beginnend bei etwa 6000 °K mit höher werdender Temperatur solange ansteigt, bis das Plasma vollionisiert ist. Die relative Druckänderung ist bei gegebenem Magnetfeld um so größer, je kleiner der Außendruck $p_A$ ist.

[1] R. WIENECKE, Z. Naturforschg. **18 a**, 1151 [1963].